ISSN 1029-9599, Physical Mesomechanics, 2018, Vol. 21, No. 2, pp. 150–156. © Pleiades Publishing, Ltd., 2018.
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A Study on Fractional Order Theory in Thermoelastic Half-Space under Thermal Loading

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Received March 14, 2017

Abstract—In this study, the effect of fractional order derivative on a two-dimensional problem due to thermal shock with weak, normal and strong conductivity is established. The governing equations are taken in the context of Green and Naghdi of type III model (GNIII model) under fractional order derivative. Based on the Laplace and exponential Fourier transformations with eigenvalues approach, the analytical solutions has been obtained. For weak, normal and strong conductivity, the numerical computations for copper-like medium have been done and the results are shown numerically. The graphical results indicate that the effect of fractional order parameter has a major role on all physical quantities involved in the problem.

DOI: 10.1134/S102995991802008X

Keywords: eigenvalue approach, Green and Naghdi of type III model, fractional order generalized thermoelasticity

1. INTRODUCTION

Many existing models of physical processes have been modified successfully by using the fractional calculus. A series of integral theories and fractional derivatives was created in the last half of the last century. Various approaches and definitions of fractional derivatives have become the main object of numerous studies [1, 2]. Recently, to investigate the anomalous diffusion, a considerable research effort has been expended, which is characterized by the fractional time equation of wave diffusion by Kimmich [3] as in the form below

$$\rho c = kI^{\alpha} c_{,ii}, \ i = 1, 2, 3, \tag{1}$$

where k is the diffusion conductivity, ρ is the mass density, c is the concentration and I^{α} is the fraction of Riemann–Liouville integral operator of order α. It introduced as a natural generalization of the well-known integral $I^{\alpha} f(t)$ repeated m times and wrote in the form of convolution type [4]

$$I^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(\alpha)} \displaystyle\int_0^t (t-\xi)^{\alpha} f(\xi) \mathrm{d}\xi, \ 0 < \alpha \le 2, \\ f(t), \ \alpha = 0, \end{cases} \tag{2}$$

where $\Gamma(\alpha)$ is the Gamma function. The fractional order of weak, normal and strong heat conductivity under generalized thermoelastic theory was applied by Youssef [5] in the following form

$$q_i + \tau_0 \frac{\partial q_i}{\partial t} = -KI^{\alpha-1} \nabla T, \ 0 < \alpha < 2. \tag{3}$$

By using Taylor expansion of time-fractional order, Ezzat and El-Karamany [6] proposed a new fractional order generalized thermoelasticity model, which developed by Jumarie [7] as

$$q_i + \frac{\tau_0^{\alpha}}{\Gamma(\alpha+1)} \frac{\partial^{\alpha} q_i}{\partial t^{\alpha}} = -K \nabla T, \ 0 < \alpha \le 1. \tag{4}$$

The fractional order study of generalized thermoelastic problems is an important branch in solid mechanics [8–12]. In addition, Abbas [13] studied the effects of fractional order and magnetic field in a thermoelastic medium due to moving heat source using the eigenvalue approach. Sherief and Abd El-Latief [14] studied the effect of the fractional order parameter and the variable thermal conductivity on a thermoelastic half-space. Due to thermal source, the effect of fractional order parameter on plane deformation in a thermoelastic medium was studied by Kumar et al. [15]. Abbas and Youssef [16] studied a two-dimensional thermoelastic porous material under fractional order theory. The fractional order influence in a functional graded thermoelastic material problem has been solved by Abbas [17]. Youssef and Abbas [18] studied the theory of generalized thermoelasticity with fractional order derivative in the case of variable thermal conductivity. Based upon the theory of two-temperature generalized thermoelasticity, Zenkour and Abouelregal [19] investigated the fractional heat conduction for an unbounded medium with a spherical cavity. Abbas [20] studied the solution of thermoelastic diffusion problem under fractional order theory in an infinite elastic medium with a spherical cavity.

In this work, the eigenvalue approach has been used to obtain the analytical solutions for temperature, displacement and the stress components. By employing an analytical-numerical technique based on the eigenvalues approach with Laplace and Fourier transformations, the nondimensional equations have been handled. Numerical computations for copper-like medium have been done for strong, normal and weak conductivity and the effect of the fractional order parameter has been estimated.

2. BASIC EQUATIONS

Let us consider a homogeneous, thermoelastic isotropic half-space $y \geq 0$ at initial uniform temperature T_0. Cartesian coordinate system (x, y, z) has been used with y axis is taken perpendicular to the bounding plane (Fig. 1). The displacement vector has the form $\mathbf{u} = (u, v, 0)$. The governing equations have the following form

$$(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2}+(\lambda+\mu)\frac{\partial^2 v}{\partial x\partial y}+\mu\frac{\partial^2 u}{\partial y^2}-\gamma\frac{\partial T}{\partial x}=\rho\frac{\partial^2 u}{\partial t^2}, \tag{5}$$

$$(\lambda+2\mu)\frac{\partial^2 v}{\partial y^2}+(\lambda+\mu)\frac{\partial^2 u}{\partial x\partial y}+\mu\frac{\partial^2 v}{\partial x^2}-\gamma\frac{\partial T}{\partial y}=\rho\frac{\partial^2 v}{\partial t^2}, \tag{6}$$

$$I^{\alpha-1}\left(K^*+K\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\frac{\partial^2}{\partial t^2}\left(\rho c_{\mathrm{e}}T+\gamma T_0\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right), \tag{7}$$

where the operator of fractional integral can be defined as the following [21]:

$$I^{\alpha}f(\varphi)=\frac{1}{\Gamma(\alpha)}\int_0^{\varphi}(\varphi-\varepsilon)^{\alpha-1}f(\varepsilon)\,\mathrm{d}\varepsilon, \tag{8}$$

$$\begin{cases}0<\alpha<1 & \text{for weak conductivity,}\\ \alpha=1 & \text{for normal conductivity,}\\ 1<\alpha\leq 2 & \text{for strong conductivity,}\end{cases}$$

$$\sigma_{xx}=(\lambda+2\mu)\frac{\partial u}{\partial x}+\lambda\frac{\partial v}{\partial y}-\gamma(T-T_0), \tag{9}$$

$$\sigma_{yy}=(\lambda+2\mu)\frac{\partial v}{\partial y}+\lambda\frac{\partial u}{\partial x}-\gamma(T-T_0), \tag{10}$$

$$\sigma_{xy}=\mu\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right), \tag{11}$$

where λ and μ are the elastic parameters, T is the increment of temperature, ρ is the density of mass, σ_{xx}, σ_{xy} and σ_{yy} are the stress components, T_0 is the body reference temperature, c_{e} is the specific heat at constant strain, K is the thermal conductivity, $\gamma = (2\lambda + 3\mu)\times \alpha_{\mathrm{t}}$, and α_{t} is the linear thermal expansion coefficient. For convenience, the nondimensional variables can be introduced on the following form:

$$\begin{gathered}(u', v, x', y')=\frac{c}{\varsigma}(u, v, x, y),\ T'=\frac{T-T_0}{T_0},\\ (\sigma'_{xx}, \sigma'_{yy}, \sigma'_{xy})=\frac{1}{\mu}(\sigma_{xx}, \sigma_{yy}, \sigma_{xy}),\ t=\frac{c^2}{\varsigma}t,\end{gathered} \tag{12}$$

where $c^2=(\lambda+2\mu)/\rho$, $\varsigma=K/(\rho c_{\mathrm{e}})$.

In terms of these nondimensional variables (12), Eqs. (5)–(11), after suppressing the primes, can be written as

$$\beta\frac{\partial^2 u}{\partial x^2}+(\beta-1)\frac{\partial^2 v}{\partial x\partial y}+\frac{\partial^2 u}{\partial x^2}-\varpi\frac{\partial T}{\partial x}=\beta\frac{\partial^2 u}{\partial t^2}, \tag{13}$$

$$\beta\frac{\partial^2 v}{\partial y^2}+(\beta-1)\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}-\varpi\frac{\partial T}{\partial y}=\beta\frac{\partial^2 v}{\partial t^2}, \tag{14}$$

$$I^{\alpha-1}\left(\varepsilon_1+\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)=\left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right)\left(T+\varepsilon_2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right), \tag{15}$$

$$\sigma_{xx}=\beta\frac{\partial u}{\partial x}+(\beta-2)\frac{\partial v}{\partial y}+\frac{\partial^2 u}{\partial x^2}-\varpi T, \tag{16}$$

$$\sigma_{yy}=\beta\frac{\partial v}{\partial y}+(\beta-2)\frac{\partial u}{\partial x}-\varpi T, \tag{17}$$

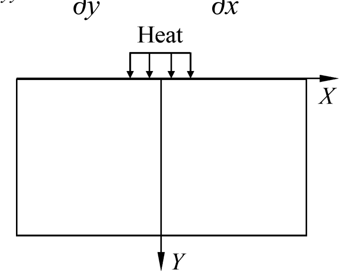

Fig. 1. Geometry of the problem.